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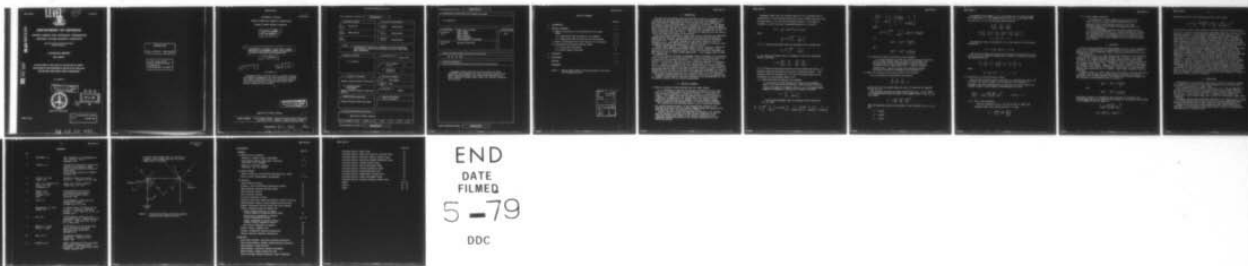
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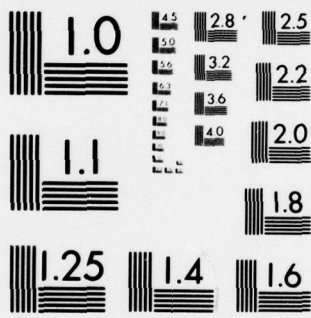
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TECHNICAL REPORT

WSRL-0030-TR

CALCULATION OF THE RADII OF CURVATURE OF SHORT-
WAVELENGTH ELECTROMAGNETIC WAVES AFTER SPECULAR
REFLECTION FROM SIMPLE SHAPE CONDUCTORS

L.A. NICHOLLS

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10 L.A./Nicholls

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S U M M A R Y

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Deschamps' matrix relations for the curvature of short-wavelength electromagnetic rays when specularly reflected from smooth conducting surfaces are reviewed and applications of these relations to double-bounce radar cross section estimation are discussed.

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Figure 1. Typical double-bounce reflection geometry associated with diagonal matrices.

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1. INTRODUCTION

This note outlines methods which are described in detail in reference 1, for calculating the curvature characteristics of a reflected electromagnetic wave, given the curvature characteristics of the incident wave and a geometrical description of the reflecting surface at the point of reflection. The methods are based on geometrical optics and are of particular use in estimating double-bounce radar cross sections, for which individual radii of curvature of reflected waves are required, when the radii of curvature of the incident wave for the second reflecting surface are finite. This work has application in the calculation of the radar cross section of an aircraft.

Since the standard textbooks on radar cross section estimation were published, simpler methods than Fock's tensor method, (ref.3,6) have been presented for determining the curvature characteristics of reflected waves for the general case, (ref.1,7,8). The prime reference for the simpler method is that of Deschamps, (ref.1) with application of the formulae for specific situations being outlined by Kouyoumjian and Pathak (ref.7), and Lee, (ref.8).

Although simple examples of Deschamps general "mirror" relations, (which are the basis of the method outlined below), appear in other references, either explicitly, (ref.9) or implicitly, (ref.5), it was felt that their usefulness and applicability were not widely enough appreciated. This document aims to widen this appreciation.

Use of the "mirror" relations of Deschamps requires knowledge of the principal radii of curvature of the reflecting surface at the reflection point and of the orientation of the principal directions. In specific instances, such as surfaces of revolution, these may be readily determined sometimes by inspection (ref.2). However for a general point on a general body such as an ellipsoid (which is frequently employed for the local representation of aircraft surfaces), use of relations from three dimensional co-ordinate geometry is required (ref.10). Appropriate formulae which are set down below, have been programmed on a digital computer (ref.11), for ease of solution of the magnitude of the principal radii of curvature and principal directions for a specified point on a given ellipsoid.

The methods outlined below have been verified in part (ref.2) by applying them to a number of double-bounce situations considered in the textbooks (ref.3), (viz. two adjacent spheres, and two adjacent paraboloids).

2. OUTLINE OF METHODS

2.1 Radii of curvature of the reflected wave for simple shapes

It is convenient to describe the curvature characteristics of a wavefront by its curvature matrix Q^T , which describes the way in which the total wavefront curvature is divided with respect to a specified axial system. For example, when the two principal directions of curvature of a wavefront are parallel to two lateral axes of the specified axial system, the curvature matrix is diagonal, with the diagonal elements being the inverse of the principal radii of curvature of the wavefront.

In a similar way, the curvature matrix, C_0 for a reflecting surface, for a given reflection point, is also diagonal when referred to its principal directions, with the diagonal elements being the inverse of the local principal radii of curvature of the surface.

Clearly, the way in which the components of curvature of an incident wave are modified on reflection by the curvature properties of the reflecting surface must depend on the relative orientation of the respective principal directions of the incident wavefront and of the reflecting surface.

Deschamps showed that the curvature matrix Q^r of a reflected wave is related(ref.1) to the curvature matrix Q_o^i of the incident wave and to the curvature matrix C_o of the local reflecting surface, to the angle of incidence θ_i and to the axial transformation matrix G as follows:

$$Q^r = Q_o^i + 2(G^{-1})^T C_o G^{-1} \cos \theta_i$$

where

$$Q_o^i \cong \begin{bmatrix} 1/\rho_1^i & 0 \\ 0 & 1/\rho_2^i \end{bmatrix},$$

ρ_1^i, ρ_2^i being the principal radii of curvature of the incident wave,

$$C_o \cong \begin{bmatrix} 1/R_1 & 0 \\ 0 & 1/R_2 \end{bmatrix}$$

R_1, R_2 being the principal radii of curvature of the local reflecting surface, and where

$$G \cong \begin{bmatrix} \underline{X}_1^i \cdot \underline{U}_1 & \underline{X}_1^i \cdot \underline{U}_2 \\ \underline{X}_2^i \cdot \underline{U}_1 & \underline{X}_2^i \cdot \underline{U}_2 \end{bmatrix},$$

$\underline{X}_1^i, \underline{X}_2^i$ being the principal directions of the incident wave and $\underline{U}_1, \underline{U}_2$ being the principal directions of the local reflecting surface. The principal radii of curvature (ρ_1^r, ρ_2^r) of the reflected wave are the reciprocals of the principal values of Q^r .

2.1.1 Application when the matrices are diagonal

In the case of a wave incident on a surface with incidence angle θ_i and with one of the surface's local principal directions lying in the plane of incidence, figure 1, as in two double-bounce examples of reference 3, then matrix G becomes as follows:

$$G = \begin{bmatrix} 1 & 0 \\ 0 & \cos \theta_i \end{bmatrix}$$

If the local principal radii of curvature of the surface are R_1, R_2 , then

$$Q^r = \begin{bmatrix} 1/\rho_1^i & 0 \\ 0 & 1/\rho_2^i \end{bmatrix} + 2 \cos \theta_i \begin{bmatrix} 1 & 0 \\ 0 & 1/\cos \theta_i \end{bmatrix} \begin{bmatrix} 1/R_1 & 0 \\ 0 & 1/R_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1/\cos \theta_i \end{bmatrix}$$

$$= \begin{bmatrix} 1/\rho_1^i & 0 \\ 0 & 1/\rho_2^i \end{bmatrix} + \begin{bmatrix} \frac{2 \cos \theta_i}{R_1} & 0 \\ 0 & \frac{2}{R_2 \cos \theta_i} \end{bmatrix}$$

where

$$Q^r \cong \begin{bmatrix} 1/\rho_1^r & 0 \\ 0 & 1/\rho_2^r \end{bmatrix} \quad \text{when } Q^r \text{ is diagonal}$$

i.e.

$$1/\rho_1^r = 1/\rho_1^i + \frac{2 \cos \theta_i}{R_1}$$

and

$$1/\rho_2^r = 1/\rho_2^i + \frac{2}{R_2 \cos \theta_i}$$

2.1.2 Application when the matrices are non-diagonal

In the more general case when the various principal directions are not either coplanar with or normal to one another, the principal radii of the reflected wave are derived from the reciprocals of the eigen values of Q^r . Specific algebraic formulae for ρ_1^r and ρ_2^r and for the associated principal directions are given by Kouyoumjian and Pathak(ref.7).

2.2 Radii of curvature of the local surface of an ellipsoid

Let the ellipsoid be described by the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

and the point on the surface where the radii of curvature are required be (x_s, y_s, z_s) .

The following relations are taken from Bell(ref.10). If the length of the perpendicular from the origin to the tangent plane at (x_s, y_s, z_s) is p , where

$$p = \left(\frac{x_s^2}{a^4} + \frac{y_s^2}{b^4} + \frac{z_s^2}{c^4} \right)^{-1/2},$$

then the direction cosines of the normal to the ellipsoid at (x_s, y_s, z_s) are

$$l = px_s/a^2$$

$$m = py_s/b^2$$

$$n = pz_s/c^2$$

The square of the lengths (r_1, r_2) of the semi-axes of section through the origin parallel to the tangent plane at (x_s, y_s, z_s) are found from the roots r_1^2, r_2^2 of the following equation(ref.10):

$$r^4 \left(\frac{\ell^2}{b^2 c^2} + \frac{m^2}{a^2 c^2} + \frac{n^2}{a^2 b^2} \right) - r^2 \left\{ \left(\frac{1}{b^2} + \frac{1}{c^2} \right) \ell^2 + \left(\frac{1}{c^2} + \frac{1}{a^2} \right) m^2 + \left(\frac{1}{a^2} + \frac{1}{b^2} \right) n^2 \right\} + \ell^2 + m^2 + n^2 = 0$$

The principal radii of curvature R_1, R_2 of the surface of the ellipsoid are(ref.10):

$$R_1 = r_1^2/p \text{ and } R_2 = r_2^2/p$$

The direction cosines describing the two principal directions $(\lambda_1, \mu_1, \nu_1)$ and $(\lambda_2, \mu_2, \nu_2)$ are found by substituting r_1^2 and r_2^2 in turn in the following relations(ref.10):

$$\frac{\lambda}{\ell} \cdot \left(\frac{r^2}{a^2} - 1 \right) = \frac{\mu}{m} \cdot \left(\frac{r^2}{b^2} - 1 \right) = \frac{\nu}{n} \cdot \left(\frac{r^2}{c^2} - 1 \right)$$

where

$$\lambda_i^2 + \mu_i^2 + \nu_i^2 = 1, \quad i = 1, 2.$$

2.3 Calculation of reflected electric fields

Once the two principal radii of curvature of a reflected wave (ρ_1^r, ρ_2^r) are known, the relative value E_r of the electric field voltage at a distance R from the reflection point in free space for unity value of the incident electric field voltage is obtained(ref.3,4,5) from:

$$\left| \frac{E_r}{E_i} \right| = \left\{ \frac{\rho_1^r \rho_2^r}{(\rho_1^r + R)(\rho_2^r + R)} \right\}^{1/2}, \quad \text{with } E_i = 1.$$

2.3.1 Use in RCS calculation

The above relation for $|E_r / E_i|$ can be used to obtain the resultant radar cross section (σ) as $R \rightarrow \infty$ from

$$\sigma = \lim_{R \rightarrow \infty} 4\pi R^2 \cdot \left| \frac{E_r}{E_i} \right|^2$$

2.3.2 Double bounce situations

Alternatively, if double reflections are involved, the above relation for $|E_r/E_i|$ enables determination of the magnitude of the field voltage incident at the second reflection point by substituting the appropriate range R in the above expression.

For use of Deschamps' relations for the curvature matrix at the second reflection point, it is clearly necessary to increase the calculated radii of curvature of the first reflected ray by the separation of the two reflection points to obtain the radii of curvature of the second incident ray at the second reflection point, (ref.2).

3. DISCUSSION

In most scattering problems utilizing geometric optics, a basic computation step is the determination of the reflected field for a specified point of reflection on a given conducting surface, knowing the characteristics of the incident wavefront.

In the general case, (i.e. when the principal directions of the scattering surface are not necessarily either coplanar with or normal to the plane of incidence), the standard textbooks on radar scattering from simple shapes (ref.3,4,5), fall back on Fock's method (ref.6) which uses tensor notation and was originally published in 1950. Reference 3 gives a detailed description and examples of the method. However, the required calculation in the general case is more readily handled by Deschamps' relations (ref.1), using the more familiar matrix notation, as it is indeed in some simple situations now to be discussed.

As noted in para 2.1.1 above, when one of the surface's principal directions is coplanar with the plane of incidence which also contains a principal direction of the incident wavefront the relevant matrix becomes diagonal and the curvatures of the reflected waves are given by:

$$1/\rho_1^r = 1/\rho_1^i + \frac{2 \cos \theta_i}{R_1}$$

and

$$1/\rho_2^r = 1/\rho_2^i + \frac{2}{R_2 \cos \theta_i}$$

From para 2.3 above, at a distance R from the point of reflection, the relative magnitude of the reflected field voltage to the incident field voltage at the reflection point is equal to:

$$\left\{ \frac{\rho_1^r \rho_2^r}{(\rho_1^r + R)(\rho_2^r + R)} \right\}^{1/2}$$

$$= \{ (1 + R/\rho_1^r) (1 + R/\rho_2^r) \}^{-1/2}$$

Substitution of the above relations for $1/\rho_1^r$, $1/\rho_2^r$ gives:

$$\left\{ \left(1 + \frac{R}{\rho_1^i} + \frac{2R \cos \theta_i}{R_1} \right) \left(1 + \frac{R}{\rho_2^i} + \frac{2R}{R_2 \cos \theta_i} \right) \right\}^{-1/2}$$

In the case of a point source located at distance d from the reflecting point, $\rho_1^i = \rho_2^i = d$ and the above expression then becomes identical in form, (but with different notation), to that quoted by Bowman et al (ref.5, pg 24). This latter relation is used extensively, but without derivation in reference 5, since many of the scattering examples considered there relate to surfaces of revolution with the plane of incidence containing the axis of revolution and hence containing a principal direction of the surface at the reflection point. It is seen that Deschamps' relations readily provide the necessary relations for such simple situations, as in fact they do for more complex situations.

The geometrical description of common scattering surfaces also differs somewhat between authors when utilizing methods of geometrical optics for the solution of scattering problems. Thus Bowman et al (ref.5) utilize oblate spheroidal co-ordinates rather than Cartesian co-ordinates when describing high frequency scattering from a conducting oblate spheroid, (an ellipsoid of revolution). The choice of such co-ordinates enables the high frequency solution to be considered as a special case of the solution for an arbitrary frequency. However, when prime interest is in a geometrical optics solution, the use of Cartesian co-ordinates and the standard methods of co-ordinate geometry (ref.10) would appear to provide the simplest methods for most people.

Although the techniques outlined above, based on Deschamps' relations, are satisfactory for smooth conductors of finite (but non-zero) radii of curvature, there is also a need in practice for similar simple techniques by which to describe specular reflections from bodies such as finite cylinders of length L , having one infinite radius of curvature.

4. CONCLUSIONS

Deschamps (ref.1) has provided a simple matrix relation by which to compute the components of curvature of short electromagnetic waves following their reflection from conductors of simple shape.

Such information directly enables calculation of the relative magnitude of the specularly reflected field at a given distance from a simple conductor, provided its principal radii of curvature at the reflection point are finite but non-zero. The relation used twice in sequence readily enables the calculation of double-bounce radar cross sections for two adjacent simple shape conductors.

Deschamps' relations were published subsequent to the standard books on radar cross section estimation (ref.3,4,5) which quote the more complicated tensor relations published by Fock in 1950 to meet the same requirement (ref.6).

Because of the generally greater familiarity with matrices than tensors, Deschamps' relations present a significant advance in means of estimating the characteristics of reflected waves.

NOTATION

| | |
|--|---|
| C_o | Curvature matrix of a local reflecting surface |
| E_i | Voltage of incident field |
| E_r | Voltage of reflected field |
| G | Transformation matrix connecting the principal directions of the reflecting surface and those of the incident wavefront |
| L | Length of cylinder |
| Q_o^i | Matrix describing the components of curvature of an incident wavefront |
| Q^r | Matrix describing the components of curvature of a reflected wavefront |
| R | Range from reflecting point |
| $\left. \begin{matrix} R_1 \\ R_2 \end{matrix} \right\}$ | Radii of curvature of a local reflecting surface |
| $\left. \begin{matrix} \underline{U}_1 \\ \underline{U}_2 \end{matrix} \right\}$ | Unit vectors describing the principal directions of the local reflecting surface |
| $\left. \begin{matrix} \underline{X}_1 \\ \underline{X}_2 \end{matrix} \right\}^i$ | Unit vectors describing the principal directions of the incident wavefront |
| $\left. \begin{matrix} a \\ b \\ c \end{matrix} \right\}$ | Lengths of semi-axes of an ellipsoid |
| d | Separation of two reflection points, or distance of a point source from a reflection point |
| $\left. \begin{matrix} l \\ m \\ n \end{matrix} \right\}$ | Direction cosines of local normal to an ellipsoid |
| p | Length of perpendicular from the origin to a tangent plane |
| $\left. \begin{matrix} r_1 \\ r_2 \end{matrix} \right\}$ | Lengths of semi axes of an elliptical section |
| $\left. \begin{matrix} x \\ y \\ z \end{matrix} \right\}$ | General Cartesian co-ordinates |
| $\left. \begin{matrix} x_s \\ y_s \\ z_s \end{matrix} \right\}$ | Co-ordinates of a point of reflection |

| | |
|--|---|
| θ_i | Angle of incidence |
| λ | Wavelength of electromagnetic radiation |
| $\left. \begin{array}{l} \lambda_i \\ \mu_i \\ \nu_i \end{array} \right\}$ | Direction cosines of the principal directions of a local reflecting surface, $i = 1, 2$ |
| $\left. \begin{array}{l} \rho_1^i \\ \rho_2^i \end{array} \right\}$ | Principal radii of curvature of an incident wavefront |
| $\left. \begin{array}{l} \rho_1^r \\ \rho_2^r \end{array} \right\}$ | Principal radii of curvature of a reflected wavefront |
| σ | Radar cross section |

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R_2 and R_4 are principal radii of curvature of surfaces 1 and 2 respectively and lie in the common plane of incidence.

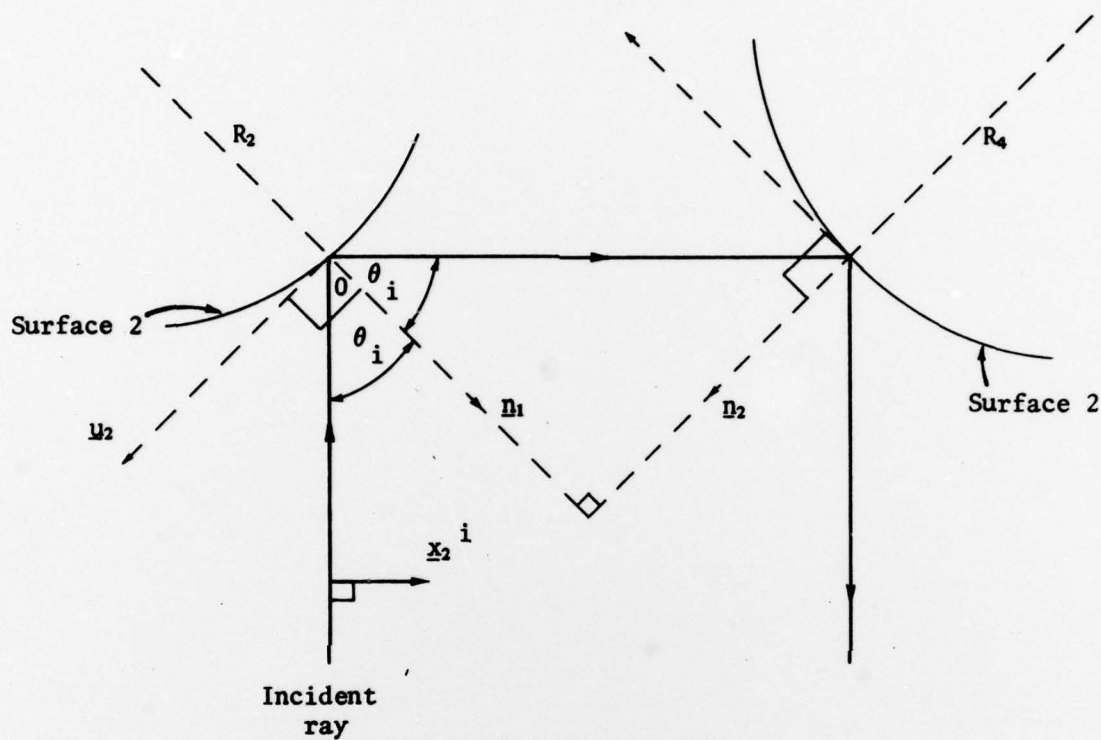


Figure 1. Typical double-bounce reflection geometry associated with diagonal matrices

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